GEOMETRICALLY NON-LINEAR MODELLING OF SHALLOW FOUNDATION COLLAPSE ON SLOPING GROUND

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Summary. This paper investigates the behaviour of shallow foundations on sloping ground when loaded up to collapse, in order to assess the effect of including geometrical non-linearity in the finite element model. A comparison is presented between the collapse loads and mechanisms obtained with either small strain or large strain formulations, for flexible strip footings with centred vertical loading on a cohesive soil for undrained conditions. The formulas proposed by several authors to account for the sloping ground effect and the footing depth effect on bearing capacity are used to appraise the numerical results.

1 INTRODUCTION

The bearing capacity of shallow foundations depends on a number of factors further to the soil resistance parameters, such as the shape, depth and tilt of the foundation base, the inclination and eccentricity of the applied load, as well as the slope of the ground, to name but a few.

The problem has been studied by Terzaghi¹ and later developed by several authors²⁻⁴. The basic bearing capacity equation corresponds to the case of a flexible strip footing with horizontal base of width **B**, located at depth **D** in a homogeneous and horizontal soil stratum with unit weight γ , cohesion c and friction angle ϕ' (Figure 1). If a vertical, centred load **Q** is applied, the bearing capacity q_{ut} is:

$$\boldsymbol{q}_{ult} = \boldsymbol{Q}_{ult} / \boldsymbol{B} = \boldsymbol{c} \ \boldsymbol{N}_c + \boldsymbol{\bar{q}} \ \boldsymbol{N}_q + 1/2 \, \boldsymbol{B} \ \boldsymbol{\gamma} \boldsymbol{N}_{\boldsymbol{\gamma}}$$
(1)

The bearing capacity factors N_c , N_a and N_r , are given by⁵:

$$N_{c} = (N_{q} - 1)/\tan\phi' \qquad N_{q} = e^{\pi \tan\phi'} \tan^{2}\left(\pi/4 + \phi'/2\right) \qquad N_{\gamma} = 2(N_{q} - 1)\tan\phi' \tag{2}$$

The shearing resistance of the soil layer of thickness D above the base of the footing is usually neglected, its effect being considered merely as that of a surcharge \bar{q} .

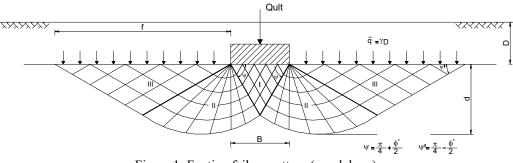


Figure 1: Footing failure pattern (rough base)

2 SLOPING GROUND SURFACE EFFECT

When the ground surface below which the footing is located makes an angle β with respect to the horizontal, the corrective factors g_c , g_q and g_{γ} are introduced in Equation (1) which becomes:

$$\boldsymbol{q}_{ult} = \boldsymbol{Q}_{ult} / \boldsymbol{B} = \boldsymbol{c} \ N_c \boldsymbol{g}_c + \boldsymbol{\bar{q}} \ N_q \boldsymbol{g}_q + 1/2 \, \boldsymbol{B} \, \boldsymbol{\gamma} N_{\boldsymbol{\gamma}} \boldsymbol{g}_{\boldsymbol{\gamma}}$$
(3)

Those dimensionless factors are defined as⁶:

$$\boldsymbol{g}_{c} = \boldsymbol{g}_{q} - (1 - \boldsymbol{g}_{q}) / (N_{c} \tan \phi') \qquad \boldsymbol{g}_{q} = \boldsymbol{g}_{\gamma} = (1 - \tan \beta)^{2}$$
(4)

When $\phi' = 0$ we have for g_c (with β expressed in radians)

$$\boldsymbol{g}_{c} = 1 - 2\boldsymbol{\beta} / (\boldsymbol{\pi} + 2) \tag{5}$$

3 EFFECT OF THE SHEARING RESISTANCE OF THE OVERBURDEN

The contribution to the bearing capacity of the shearing resistance of the soil above the base of the footing may be accounted for by introducing in Equation (1) the following depth corrective factors d_c , d_a and d_y , proposed by Brinch Hansen³ for the case of $D/B \le 1$:

$$d_{c} = d_{q} - (1 - d_{q}) / (N_{c} tan \phi') \qquad d_{q} = 1 + 2 tan \phi' (1 - sin \phi')^{2} D / B \qquad d_{\gamma} = 1$$
(6)

When $\phi' = 0$ we have for d_c

$$d_{c} = 1 + 2D/[(\pi + 2)B] \approx 1 + 0.4D/B$$
(7)

4 NUMERICAL STUDY

In order to check the effect on the bearing capacity of the ground surface slope the five 8noded element meshes of Figure 2 have been used, with β values of 0°, 15°, 25°, 30° and 45°, respectively. A purely cohesive weightless soil was considered, with E=12MPa, v=0.49 and c_u=30kPa. Figure 3 shows the small strain results of the evolution of d_y/B with q/c_u , d_y being the vertical displacement at the centre of the footing and q the applied vertical pressure. The agreement of the collapse load with the analytical solution is excellent for all the values considered for the ground surface slope β . The large strain results of Figure 4 agree well with the analytical solution now modified to include the depth factor of Equation (7) with $D=d_y$, reflecting the positive effect of the progressive embedment of the foundation base.

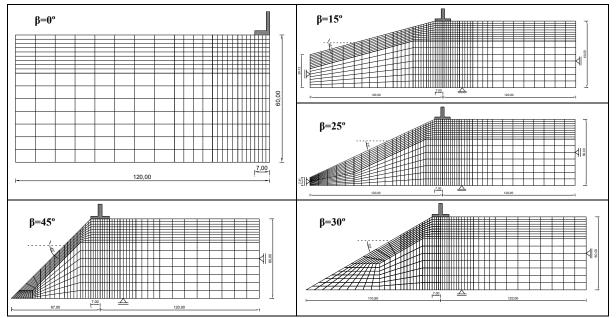
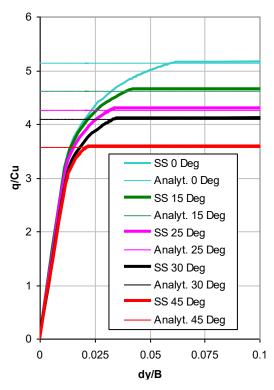


Figure 2: Finite element meshes





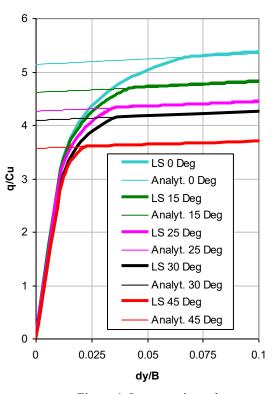
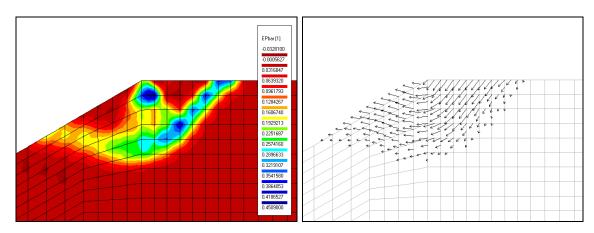


Figure 4: Large strain analyses



In Figure 5 are shown the effective strain and displacement vector plots for the large strain analysis of the β =30° case.

Figure 5: Effective plastic strain and displacement pattern for β =30° (large strain analysis)

4 CONCLUSIONS

The influence of the ground surface slope on the bearing capacity of strip footings on purely cohesive soil has been investigated. The small strain analyses match well the analytical collapse loads, while the consideration of large strains is instrumental to capture the contribution to the bearing capacity of the shearing resistance of the soil above the foundation base as it moves progressively downwards with increasing applied load.

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